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Some Hypergeometric Generalizations of Question-700 of S. Ramanujan, K.R. Rama Aiyar and K. Appukuttan Erady

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ABSTRACT: In this paper we obtained some interesting truncated Gaussian hypergeometric summation theorems whose arguments, numerator and denominator parameters are real numbers. A known results of S. Ramanujan, K.R. Rama Aiyar and K. Appukuttan Erady also obtained as a special case of our findings.

Keywords: Gamma function, Pochhammer symbol, Generalized hypergeometric function of one variable, Truncated hypergeometric series, Euler's Identity.

I. INTRODUCTION

The Pochhammer symbol or generalized factorial function or shifted factorial is defined by

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = \begin{bmatrix} 1; n=0 \\ a(a+1)(a+2)...(a+n-1); n=1,2,3... \end{bmatrix} \tag{1}$$

Where $a \neq 0,-1,-2,...$ and the notation 'Γ' stands for Gamma function.

$$[(a_A)]_n = (a_1)_n (a_2)_n ... (a_A)_n = \prod_{m=1}^{A} (a_m)_n = \prod_{m=1}^{A} \frac{\Gamma(a_m+n)}{\Gamma(a_m)} \tag{2}$$

Where $a_1, a_2, ..., a_A, b_1, b_2, ..., b_B$ and z may be real and complex numbers.

$$(a)_{-n} = \frac{\Gamma(a-n)}{\Gamma(a)} = \frac{(-1)^n}{(1-a)_n} \tag{3}$$

Where $a \neq ...-3,-2,-1,0,1,2,3,...$ (4)

and $n = 1,2,3,...$ (5)

Non Terminating Gaussian Hypergeometric Series: The generalized hypergeometric function of one variable is defined as in Prudnikov (1986) [1, p.437]:

$${}_A F_B \left[\begin{matrix} a_1, a_2, ..., a_A; \\ b_1, b_2, ..., b_B; \end{matrix} z \right] = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n ... (a_A)_n}{(b_1)_n (b_2)_n ... (b_B)_n} \frac{z^n}{n!} \text{ OR } {}_A F_B \left[\begin{matrix} (a_A); \\ (b_B); \end{matrix} z \right] = \sum_{n=0}^{\infty} \frac{[(a_A)]_n}{[(b_B)]_n} \frac{z^n}{n!} \tag{6}$$

Where are neither zero nor negative integers.

Terminating Gaussian Hypergeometric Series: The terminating generalized hypergeometric function of one variable is defined as

$${}_{A+1} F_B \left[\begin{matrix} -n, (a_A); \\ (b_B); \end{matrix} z \right] = \sum_{n=0}^{\infty} \frac{(-n)_n (a_1)_n (a_2)_n ... (a_A)_n}{(b_1)_n (b_2)_n ... (b_B)_n} \frac{z^n}{n!} \tag{7}$$

Where n is non-negative integer and are neither zero nor negative integers.

Truncated Gaussian Hypergeometric Series: The terminating generalized hypergeometric function of one variable is defined as

$${}_A F_B \left[\begin{matrix} (a_A); \\ (b_B); \end{matrix} z \right]_n = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n ... (a_A)_n}{(b_1)_n (b_2)_n ... (b_B)_n} \frac{z^n}{n!} \tag{8}$$

Where are neither zero nor negative integers and the suffix '*n*' indicates that only first (n+1) terms of F series are to be included in the expansion, given in slater (1966) [2, pp.83-84(2.6.1.1, 2.6.1.7, 2.6.1.9)].

Lemma: If a, p and n are suitably adjusted real or complex numbers such that with associated pochhammer's symbols we can defined

$$(a+kp)=\frac{a\left(\frac{a+p}{p}\right)_k}{\left(\frac{a}{p}\right)_k} \qquad \forall \quad k=0,1,2,3,\ldots.. \tag{9}$$

Proof: By the definition of pochhammer's symbol $(a)_k=\frac{\Gamma(a+k)}{\Gamma(a)}$ and using the recurrence relation $\Gamma(z+1)=z\,\Gamma(z)$, then we take R.H.S,

$$\frac{a\left(\frac{a+p}{p}\right)_k}{\left(\frac{a}{p}\right)_k}=\frac{a\,\Gamma\left(\frac{a+p}{p}+k\right)\Gamma\left(\frac{a}{p}\right)}{\Gamma\left(\frac{a+p}{p}\right)\Gamma\left(\frac{a}{p}+k\right)}$$

$$=\frac{a\,\Gamma\left(1+\frac{a}{p}+k\right)\Gamma\left(\frac{a}{p}\right)}{\Gamma\left(1+\frac{a}{p}\right)\Gamma\left(\frac{a}{p}+k\right)}$$

$$=\frac{a\left(\frac{a}{p}+k\right)\Gamma\left(\frac{a}{p}+k\right)\Gamma\left(\frac{a}{p}\right)}{\left(\frac{a}{p}\right)\Gamma\left(\frac{a}{p}\right)\Gamma\left(\frac{a}{p}+k\right)}$$

$$=(a+kp)$$

This is L.H.S.

It should be remarked that Ramanujan has no notation for hypergeometric series [9 and 10,(part-II),p.8]. All formulas are stated by writing out the first few terms in each series.

R.P. Agarwal [1] gave the following correct forms of Slater's theorems for truncated hypergeometric function.

$${}_2F_1\left[\begin{matrix}a_0,a_1;\\1+b_1;\end{matrix}\;1\right]_N=\frac{(1+a_0)_N(1+a_1)_N}{(1+b_1)_N\;N!}$$

Where $a_0+a_1=b_1$

$${}_3F_2\left[\begin{matrix}a_0,a_1,a_2;\\1+b_1,1+b_2;\end{matrix}\;1\right]_N=\frac{(1+a_0)_N(1+a_1)_N(1+a_2)_N}{(1+b_1)_N\,(1+b_2)_N\;N!}$$

Subject to the condition:

$a_0+a_1+a_2=b_1+b_2$

$a_0a_1+a_1a_2+a_0a_2=b_1b_2$

$a_0a_1a_2\neq 0$

$${}_4F_3\left[\begin{matrix}a_0,a_1,a_2,a_3;\\1+b_1,1+b_2,1+b_3;\end{matrix}\;1\right]_N=\frac{(1+a_0)_N(1+a_1)_N(1+a_2)_N(1+a_3)_N}{(1+b_1)_N\,(1+b_2)_N\,(1+b_3)_N\,N!}$$

Subject to the condition:

$a_0+a_1+a_2+a_3=b_1+b_2+b_3$

$a_0a_1+a_0a_2+a_0a_3+a_1a_2+a_1a_3+a_2a_3=b_1b_2+b_1b_3+b_2b_3$

$a_0a_1a_2+a_0a_1a_3+a_0a_2a_3+a_1a_2a_3=b_1b_2b_3$

Lemma: If a and b are parameters and $c\neq 1$ then

$$\lim_{c\to 1}\frac{\left(\frac{(b+2)-(a+2)c^{\frac{1}{p}}}{1-c^{\frac{1}{p}}}\right)_k}{\left(\frac{(b+1)-(a+1)c^{\frac{1}{p}}}{1-c^{\frac{1}{p}}}\right)_k}=1 \tag{10}$$

Proof:

Consider the L.H.S

$$=\lim_{c\to 1}\frac{\left(\frac{(b+1)-(a+1)c^{\frac{1}{p}}+1-c^{\frac{1}{p}}}{1-c^{\frac{1}{p}}}\right)_k}{\left(\frac{(b+1)-(a+1)c^{\frac{1}{p}}}{1-c^{\frac{1}{p}}}\right)_k}$$

$$= \lim_{c\to 1} \frac{\left(\frac{(b+1)-(a+1)c^{\frac{1}{p}}}{1-c^{\frac{1}{p}}}+1\right)_k}{\left(\frac{(b+1)-(a+1)c^{\frac{1}{p}}}{1-c^{\frac{1}{p}}}\right)_k}$$

$$= \lim_{c\to 1} \frac{\frac{(b+1)-(a+1)c^{\frac{1}{p}}}{1-c^{\frac{1}{p}}}+k}{\frac{(b+1)-(a+1)c^{\frac{1}{p}}}{1-c^{\frac{1}{p}}}}$$

$$= \lim_{c\to 1} \frac{(b+1)-(a+1)c^{\frac{1}{p}}+k\left(1-c^{\frac{1}{p}}\right)}{(b+1)-(a+1)c^{\frac{1}{p}}}$$

$$= \frac{b+1-a-1+k-k}{b+1-a-1}$$

$$= \frac{b-a}{b-a} = 1$$

Which is right hand side.
Question 700 by S. Ramanujan [8,p.199], Rama Aiyar and Erady [7,p.152] see also Nagarajan and Soundararajan [5,p.15], Hardy and Wilson [4,p.331] and Rao [11,,p.48]
As an example of Ramanujan's elementry Mathematics which possesses some novelty, we refer to the question 700 posed by him in the journal of Indian Mathematical Society [8, (VII), p.199]. Sum the series

$$(a+b+1)\left\{\frac{a}{b}\right\}^2 + (a+b+3)\left\{\frac{a(a+1)}{b(b+1)}\right\}^2 + (a+b+5)\left\{\frac{a(a+1)(a+2)}{b(b+1)(b+2)}\right\}^2 \quad + \cdots n\ terms$$

to n terms. The solutions were provided by K.R. Rama Aiyar and K. Appukuttan Erady[7, VIII, p.152]. It should be remarked that Ramanujan has no notation for Hypergeometric series stated by Berndt, Choi and Kong S-Y [2, (Part-II),p.8].
Euler's Identity: it is stated by Berndt and Rankin in on of his paper [3,p.243]

$$\sum_{k=0}^{n}\left\{(1-d_{k+1})\prod_{j=1}^{k} d_j\right\} = 1-\prod_{j=1}^{n+1} d_j$$

Where $\prod_{j=1}^{0} = 1$ (Empty product stated as unity and n is non negative integer)
Proof: Consider the L.H.S.

$$L_1 = \sum_{k=0}^{n}\left\{(1-d_{k+1})\prod_{j=1}^{k} d_j\right\} = (1-d_1)\prod_{j=1}^{0} d_j + (1-d_2)\prod_{j=1}^{1} d_j + (1-d_3)\prod_{j=1}^{2} d_j + (1-d_4)\prod_{j=1}^{3} d_j + \cdots$$
$$+ (1-d_n)\prod_{j=1}^{n-1} d_j + (1-d_{n+1})\prod_{j=1}^{n} d_j$$

$$= (1-d_1).1 + (1-d_2)d_1 + (1-d_3)d_1d_2 + (1-d_4)d_1d_2d_3 + \cdots + (1-d_n)d_1d_2\ldots d_{n-1}$$
$$+ (1-d_{n+1})d_1d_2\ldots d_n$$

$$= 1 - d_1 + d_1 - d_1 d_2 + d_1 d_2 - d_1 d_2 d_3 + d_1 d_2 d_3 - d_1 d_2 d_3 d_4 + \cdots + d_1 d_2 \ldots d_{n-1} - d_1 d_2 \ldots d_{n-1} d_n + d_1 d_2 \ldots d_n - d_1 d_2 \ldots d_n d_{n+1}$$

$$= 1 - d_1 d_2 \ldots d_n d_{n+1} = 1 - \prod_{j=1}^{n+1} d_j$$

This is right hand side.

II. MAIN RESULTS

$${}_4F_3\left[\begin{matrix}1, a+1, a+1, \frac{a+b+4}{2}; \\ b+2, b+2, \frac{a+b+2}{2};\end{matrix}\ 1\right]_n = \frac{(b+1)^2}{(b-a)(a+b+2)}\left[1 - \frac{(a+1)_{n+1}^2}{(b+1)_{n+1}^2}\right] \tag{11}$$

$${}_6F_5\left[\begin{matrix}1, a+1, a+1, a+1, \frac{6+B-\sqrt{B^2-12C}}{6}, \frac{6+B+\sqrt{B^2-12C}}{6}; \\ b+2, b+2, \frac{B-\sqrt{B^2-12C}}{6}, \frac{B+\sqrt{B^2-12C}}{6};\end{matrix}\ 1\right]_n = \frac{1}{(b-a)(a^2+b^2+ab+3a+3b+3)}\left[\frac{(b+1)_{n+1}^3 - (a+1)_{n+1}^3}{(b+2)_n^3}\right] \tag{12}$$

Where $B = 3a + 3b + 6$, $C = a^2 + b^2 + ab + 3a + 3b + 3$

III. DERIVATIONS

(I) Put $d_j = \frac{(a+j)^2}{(b+j)^2}$ in Euler's identity, therefore $d_{k+1} = \frac{(a+k+1)^2}{(b+k+1)^2}$, we get

$$\sum_{k=0}^{n}\left\{\left(1 - \frac{(a+k+1)^2}{(b+k+1)^2}\right)\prod_{j=1}^{k}\frac{(a+j)^2}{(b+j)^2}\right\} = 1 - \prod_{j=1}^{n+1}\frac{(a+j)^2}{(b+j)^2} \tag{13}$$

Simplify the L.H.S of (13), we get

$$= \sum_{k=0}^{n}\left\{\left(\frac{(b+k+1)^2 - (a+k+1)^2}{(b+k+1)^2}\right)\prod_{j=1}^{k}\frac{(a+j)^2}{(b+j)^2}\right\}$$

$$= \sum_{k=0}^{n}\left[\frac{(b-a)(a+b+2k+2)}{(b+k+1)^2}\right]\frac{(a+1)^2(a+2)^2(a+3)^2\ldots(a+k)^2}{(b+1)^2(b+2)^2(b+3)^2\ldots(b+k)^2}$$

$$= (b-a)\sum_{k=0}^{n}\left[\frac{(a+b+2k+2)\{(a+1)_k\}^2}{(b+k+1)^2\{(b+1)_k\}^2}\right]$$

$$= (b-a)\sum_{k=0}^{n}\left[\frac{(a+b+2k+2)\{(a+1)_k\}^2}{\{(b+1)_{k+1}\}^2}\right]$$

$$= \frac{(b-a)}{(b+1)(b+1)}\sum_{k=0}^{n}\left[\frac{(a+b+2k+2)(a+1)_k(a+1)_k}{(b+2)_k(b+2)_k}\right] \tag{14}$$

$$= \frac{(b-a)(a+b+2)}{(b+1)^2}\sum_{k=0}^{n}\left[\frac{\left(\frac{a+b+4}{2}\right)_k (a+1)_k^2\,(1)_k}{\left(\frac{a+b+2}{2}\right)_k (b+2)_k^2}\ \frac{(1)^k}{k!}\right]$$

$$= \frac{(b-a)(a+b+2)}{(b+1)^2}\ {}_4F_3\left[\begin{matrix}1, a+1, a+1, \frac{a+b+4}{2}; \\ b+2, b+2, \frac{a+b+2}{2};\end{matrix}\ 1\right]_n \tag{15}$$

Now simplify the R.H.S of (13), we get

$$= 1 - \frac{(a+1)^2(a+2)^2(a+3)^2\ldots(a+n)^2(a+n+1)^2}{(b+1)^2(b+2)^2(b+3)^2\ldots(b+n)^2(b+n+1)^2}$$

$$= 1 - \frac{(a+1)_{n+1}^2}{(b+1)_{n+1}^2} \tag{16}$$

Therefore, by equating equation (15) and (16), we get the result (11)

$${}_4F_3\left[\begin{matrix}1, a+1, a+1, \frac{a+b+4}{2}; \\ b+2, b+2, \frac{a+b+2}{2};\end{matrix}\ 1\right]_n = \frac{(b+1)^2}{(b-a)(a+b+2)}\left[1 - \frac{(a+1)_{n+1}^2}{(b+1)_{n+1}^2}\right]$$

(II) Put $d_j = \frac{(a+j)^3}{(b+j)^3}$ in Euler's identity, therefore $d_{k+1} = \frac{(a+k+1)^3}{(b+k+1)^3}$, we get

$$\sum_{k=0}^{n}\left\{\left(1 - \frac{(a+k+1)^3}{(b+k+1)^3}\right)\prod_{j=1}^{k}\frac{(a+j)^3}{(b+j)^3}\right\} = 1 - \prod_{j=1}^{n+1}\frac{(a+j)^3}{(b+j)^3} \tag{17}$$

Consider the L.H.S of equation (17)

$$= \sum_{k=0}^{n}(b-a)\{(b+k+1)^2 + (a+k-1)(b+k+1)$$

$$+ (a+k+1)^2\}\frac{(a+1)^3(a+2)^3(a+3)^3\ldots(a+k)^3}{(b+1)^3(b+2)^3(b+3)^3\ldots(b+k)^3(b+k+1)^3}$$

$$= (b-a)\sum_{k=0}^{n}\{3k^2 + (3a+3b+6)k + (a^2+b^2+ab+3a+3b+3)\}\frac{(a+1)_k^3}{(b+1)_{k+1}^3}$$

$$= 3(b-a)\sum_{k=0}^{n}\left(k + \frac{B}{6} - \frac{\sqrt{B^2-12C}}{6}\right)\left(k + \frac{B}{6} + \frac{\sqrt{B^2-12C}}{6}\right)\frac{(a+1)_k^3}{(b+1)_{k+1}^3}$$

Where $B = 3a+3b+6,\ C = a^2+b^2+ab+3a+3b+3$

Now using the identity (9), we get

$$\sum_{k=0}^{n}\left\{\left(1 - \frac{(a+k+1)^3}{(b+k+1)^3}\right)\prod_{j=1}^{k}\frac{(a+j)^3}{(b+j)^3}\right\} = \frac{3(b-a)}{(b+1)^3}\left(\frac{B}{6} - \frac{\sqrt{B^2-12C}}{6}\right)\left(\frac{B}{6} + \frac{\sqrt{B^2-12C}}{6}\right)$$

$$\times \sum_{k=0}^{n}\frac{\left(\frac{B-\sqrt{B^2-12C}+6}{6}\right)_k\left(\frac{B+\sqrt{B^2-12C}+6}{6}\right)_k}{\left(\frac{B-\sqrt{B^2-12C}}{6}\right)_k\left(\frac{B+\sqrt{B^2-12C}}{6}\right)_k}\frac{(a+1)_k(a+1)_k(a+1)_k\,(1)_k}{(b+2)_k(b+2)_k(b+2)_k}\ \frac{(1)^k}{k!}$$

$$= \frac{36(b-a)}{(b+1)^3}\,C\ {}_6F_5\left[\begin{matrix}1, a+1, a+1, a+1, \frac{6+B-\sqrt{B^2-12C}}{6}, \frac{6+B+\sqrt{B^2-12C}}{6}; \\ b+2, b+2, b+2, \frac{B-\sqrt{B^2-12C}}{6}, \frac{B+\sqrt{B^2-12C}}{6};\end{matrix}\ 1\right]_n \tag{18}$$

Where $B = 3a+3b+6,\ C = a^2+b^2+ab+3a+3b+3$

The R.H.S of equation (17), we have

$$= 1 - \frac{(a+1)^3(a+2)^3(a+3)^3\ldots(a+n)^3(a+n+1)^3}{(b+1)^3(b+2)^3(b+3)^3\ldots(b+n)^3(b+n+1)^3}$$

$$= 1 - \frac{(a+1)_{n+1}^3}{(b+1)_{n+1}^3}$$

$$= \frac{(b+1)_{n+1}^3 - (a+1)_{n+1}^3}{(b+1)_{n+1}^3} \tag{19}$$

Now equating L.H.S (18) and R.H.S (19) and using identity (5), we get

$$ {}_6F_5\left[\begin{matrix} 1, a+1, a+1, a+1, \dfrac{6+B-\sqrt{B^2-12C}}{6}, \dfrac{6+B+\sqrt{B^2-12C}}{6}; \\ b+2, b+2, b+2, \dfrac{B-\sqrt{B^2-12C}}{6}, \dfrac{B+\sqrt{B^2-12C}}{6}; \end{matrix} 1\right]_n = \frac{1}{(b-a)(a^2+b^2+ab+3a+3b+3)}\left[\frac{(b+1)_{n+1}^3-(a+1)_{n+1}^3}{(b+2)_n^3}\right] $$

Where $B = 3a+3b+6,\ \ C = a^2+b^2+ab+3a+3b+3$
This is result (12).

IV. DISCUSSION

In the derivation of (11), if we consider equation (14) and (16), we have

$$ \frac{(b-a)}{(b+1)(b+1)}\sum_{k=0}^{n}\left[\frac{(a+b+2k+2)(a+1)_k(a+1)_k}{(b+2)_k(b+2)_k}\right] = 1-\frac{(a+1)_{n+1}^2}{(b+1)_{n+1}^2} $$

Replace *a* by *a*-1, *b* by *b*-2

$$ \frac{(b-2-a+1)}{(b-1)^2}\sum_{k=0}^{n}\left[\frac{(a-1+b-2+2+2k)(a)_k(a)_k}{(b)_k(b)_k}\right] = 1-\frac{(a)_{n+1}^2}{(b-1)_{n+1}^2} $$

$$ \sum_{k=0}^{n}\left[\frac{(a+b+2k-1)(a)_k(a)_k}{(b)_k(b)_k}\right] = \frac{(b-1)^2}{(b-a-1)}\left[1-\frac{(a)_{n+1}^2}{(b-1)_{n+1}^2}\right] $$

$$ \sum_{k=0}^{n}\left[\frac{(a+b+2k-1)(a)_k^2}{(b)_k^2}\right] = \frac{1}{(b-a-1)}\left[(b-1)^2-\frac{(a)_{n+1}^2(b-1)^2}{(b-1)_{n+1}^2}\right] $$

$$ (a+b-1)\frac{(a)_0(a)_0}{(b)_0(b)_0}+\sum_{k=1}^{n}\left[\frac{(a+b+2k-1)(a)_k(a)_k}{(b)_k(b)_k}\right] = \frac{1}{(b-a-1)}\left[(b-1)^2-\frac{(a)_{n+1}^2(b-1)^2}{(b-1)_{n+1}^2}\right] $$

$$ \sum_{k=1}^{n}\left[\frac{(a+b+2k-1)(a)_k^2}{(b)_k^2}\right] = \frac{1}{(b-a-1)}\left[(b-1)^2-\frac{(a)_{n+1}^2(b-1)^2}{(b-1)^2(b)_n^2}\right]-(a+b-1) $$

$$ = \frac{1}{(b-a-1)}\left[(b-1)^2-\frac{(a)_{n+1}^2(b-1)^2}{(b-1)^2(b)_n^2}-(b-1-a)(b-1+a)\right] $$

$$ = \frac{1}{(b-a-1)}\left[(b-1)^2-\frac{(a)_{n+1}^2(b-1)^2}{(b-1)^2(b)_n^2}-(b-1)^2+a^2)\right] $$

$$ \sum_{k=1}^{n}\left[\frac{(a+b+2k-1)(a)_k^2}{(b)_k^2}\right] = \frac{1}{(b-a-1)}\left[a^2-\frac{(a)_{n+1}^2}{(b)_n^2}\right]; \quad b \neq a+1 $$

Therefore

$$ (a+b+1)\left\{\frac{a}{b}\right\}^2+(a+b+3)\left\{\frac{a(a+1)}{b(b+1)}\right\}^2+(a+b+5)\left\{\frac{a(a+1)(a+2)}{b(b+1)(b+2)}\right\}^2+\cdots n\ terms $$

$$ = \frac{1}{(b-a-1)}\left[a^2-\frac{(a)_{n+1}^2}{(b)_n^2}\right] $$

Where $b \neq a+1$.
Which is well known solution of Appu Kuttan and Rama Aiyar [4,p.331, see also 8,p.199, 7,p.152] and it is correct form of misprint result of Berndt and Rankin [3, p.242].

V. CONCLUSION

In this paper we obtained truncated hypergeometric form for ${}_4F_3$ and ${}_6F_5$. First truncated hypergeometric form for ${}_4F_3$ is used to calculate the sum the series

$$(a+b+1)\left\{\frac{a}{b}\right\}^2+(a+b+3)\left\{\frac{a(a+1)}{b(b+1)}\right\}^2+(a+b+5)\left\{\frac{a(a+1)(a+2)}{b(b+1)(b+2)}\right\}^2 \quad +\cdots n\ terms$$

It means this truncated hypergeometric form can be used for calculating the sum of other complex series. Secondly, obtained one more truncated hypergeometric form for ${}_6F_5$, that includes more numerators and denominators parameters. It generates sum of other series which was not possible by other methods.

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